Worksheet 6b

1. Let $A = \{1, 2, 3, 4\}$, give an example of a relation on A that is

   a. Reflexive and symmetric, but not transitive

   b. Reflexive and transitive, but not symmetric

   c. Symmetric and transitive, but not reflexive

   d. Transitive, but neither reflexive nor symmetric

   e. Symmetric, but neither reflexive nor transitive

2. Define a relation on $\mathbb{Z}$ by xRy if and only if $xy \geq 0$.

   Prove or disprove the following statement:

   a. R is reflexive

   b. R is symmetric

   c. R is transitive

3. Define a relation $\sim$ on $\mathbb{Z}$ by $a \sim b$ if and only if $a^2 = b^2$

   a. Prove $\sim$ is an equivalence relation on $\mathbb{Z}$

   b. Identify the distinct equivalence classes of $\mathbb{Z}/\sim$

4. Define the relation R on $\mathcal{P}(\mathbb{N})$ by ARB if and only if $A \subseteq B$

   Prove or disprove the following statements:

   (Recall: $\mathcal{P}(\mathbb{N})$ is the power set of natural numbers)

   a. R is reflexive

   b. R is symmetric

   c. R is transitive

5. Define a relation $\sim$ on $\mathbb{R}$ by $x \sim y$ if and only if $x - y \in \mathbb{Z}$

   Prove or disprove: $\sim$ is an equivalence relation on $\mathbb{R}$

6. Define a relation $\sim$ on $\mathbb{R} \times \mathbb{R}$ by $(x, y) \sim (z, w)$ if and only if $x^2 + y^2 = z^2 + w^2$

   a. Prove $\sim$ is an equivalence relation on $\mathbb{R} \times \mathbb{R}$

   b. Describe the equivalence class [(3, 4)]